MOTION OF THE TEMPERATURE FRONT AND PHASE TRANSITIONS OF A SUSPENDED IMPURITY IN FLOW THROUGH A POROUS MEDIUM

M. G. Alishaev

UDC 532.546:622.276.6

Exact solutions are obtained for two problems of nonisothermal percolation flow with phase transitions of the suspended impurity.

As a rule, formation fluids contain various impurities in dissolved form or in the crystalline state [1]. For example, Uzen and Zhetybai oils are rich in paraffin, the deposits of the Caspian basin contain sulfur, and the thermal waters of the Tarumovka deposit in the Dagestan ASSR up to 200 g/liter of mineral salts. With variation in pressure and temperature the impurities may pass from the liquid to the solid state and vice versa. The phase transitions are accompanied by the liberation (absorption) of heat, which not infrequently may have an important influence on the thermal state of the formation.

Thus, Uzen oil contains 20-35% paraffin. Under formation conditions, at a temperature of 65°C it is in the molten state, but crystallizes at temperatures of 59-62°C [2]. The density of Uzen oil is 830-860 kg/m^3, the density of the paraffin 870-920 kg/m^3. The specific heat of the oil is 2.05 kJ/(kg·K), and that of the paraffin 2.89 and 3.25 kJ/(kg·K) in the solid and liquid states respectively. The specific heat of fusion of the paraffin is 150 kJ/kg, i.e., melting 1 kg of paraffin requires about the same amount of heat as needed to heat it through 50°C. The paraffin phase transition effect should be taken into account in calculating the nonisothermal flow of oil displaced with cold water.

We will assume that in both the liquid and the solid states the impurity particles move together with the fluid and remain suspended in it. This is based on the fact that the physical velocity of the fluid in the pores v_f much exceeds the particle sedimentation rate u_0. The latter can be found from the Stokes law for spherical particle drag [3]:

$$u_0 = \frac{2\Delta\rho g R^2}{9\mu}. \tag{1}$$

As noted in [4], 69% of the paraffin particles are less than 1 μm in size. The paraffin crystals have an open structure and they accumulate to form loose shapeless clusters. The specific weight of such an enlarged particle is quite different from that of the pure paraffin and very similar to that of the dispersion medium. If for the paraffin crystals we take $\Delta\rho = 30$ kg/m^3, $R = 10^{-6}$ m, then for an oil viscosity $\mu = 4$ mPa·sec Eq. (1) gives a value of 1 μm per minute (1.5 mm per day) for the particle sedimentation rate.

The physical velocity of the fluid in the pore channels is given by

$$v_f = \frac{kG}{\mu m}. \tag{2}$$

For $m = 0.2$ and $k = 0.2$ μm^2, given a viscosity $\mu = 4$ mPa·sec and a pressure gradient $G = 4$ kPa/m, for the physical velocity we obtain the value 1 μm per second (9 cm per day). For actually realizable formation conditions the physical velocity of the fluid in the pores is approximately two orders greater than the particle sedimentation rate. Moreover, the characteristic particle sedimentation path is two orders greater than the pore radius. Accordingly, we shall assume the impurity to be suspended, and the deposition of impurity on the pore channel walls and its damping effect on percolation will not be taken into consideration.

Dagestan Polytechnic Institute, Makhachkala. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 2, pp. 248-252, February, 1987. Original article submitted November 12, 1985.

0022-0841/87/5202-0184$12.50

Below we offer solutions of two model problems which illustrate the qualitative features of the phase transitions of a suspended impurity in nonisothermal flow.

Problems of the theory of percolation of hydrocarbon mixtures with allowance for phase transitions from the liquid to the gaseous state were examined in detail in [5]. Here we will consider the phase transitions of heavy fractions from the liquid to the solid state and vice versa. The usual concept of a phase transition front (Stefan problem) is not applicable, since a front, as such, appears during crystallization but not during fusion. The impurity melts gradually, forming a transition zone, whereas crystallization is immediate, forming a moving front in which all the impurity particles solidify.

1. Let the heated fluid percolate through the cooled skeleton of the porous medium. Initially, the temperature T_0 of the skeleton and the fluid occupying the pores is kept constant and below the impurity crystallization point T_*. The injected fluid has a temperature $T_1 > T_*$. From contact with the cold skeleton the injected fluid is cooled, and after the temperature falls to the value T_*, crystallization of the impurity begins. If we disregard heat exchange with the roof and floor of the formation and neglect conductive as compared with convective heat transfer, then the temperature distribution is described by the equation [6]

$$c_* \frac{\partial T}{\partial t} + c_{\text{fl}} v \frac{\partial T}{\partial x} = 0, \ c_* = (1-m)\, c_0 + m c_{\text{fl}}. \tag{3}$$

Equation (3) signifies that temperature disturbances are propagated along the flow with the velocity

$$v_T = (c_{\text{fl}} / c_*)\, v. \tag{4}$$

However, with variation of the temperature and phase transition of the impurity c_{fl} may change significantly. This also involves a change in the propagation velocity of the temperature disturbances. In the hot zone, where the impurity has melted, the specific heat of the fluid is greater and the temperature disturbances are propagated more rapidly than in the cooled zone, where the impurity is in the solid state. At the crystallization temperature T_* the velocities of the temperature disturbances depend on the molten fraction of the impurity.

The volumetric specific heat of the fluid with the impurity can be expressed in the form:

$$\begin{gathered} c_{\text{fl}} = (1-\alpha_0)\, c_1 + \alpha_0 c_2^k, \ T < T_*, \\ c_{\text{fl}} = (1-\alpha_0)\, c_1 + \alpha c_2^p + (\alpha_0 - \alpha)\, c_2^k, \quad T = T_*, \\ c_{\text{fl}} = (1-\alpha_0)\, c_1 + \alpha_0 c_2^p, \ T > T_*. \end{gathered} \tag{5}$$

For example, for a 25% content of paraffin with a density of 880 kg/m³ in an oil with a volumetric specific heat of 1743 kJ/(m³·K) we have: $c^k_{\text{fl}} = 1943$, $c^k_* = 2022$, $c^p_* = 2021$, $c^p_{\text{fl}} = 2036$ kJ/(m³·K). Phase transition has little effect on the volumetric specific heat of the formation ($m = 0.2$), whereas the effect of the liquid is appreciable. For the velocity of the temperature disturbances we have $v^p_T = 0.9931v$ and $v^k_T = 0.9614v$ for the heated and cooled zones respectively.

On the injection of a hot fluid the temperature disturbances proceeding from the inlet will reach the cooled zone of the formation, forming a thermal front with a temperature jump at $x = x_*(t)$. The solution of the problem is written in the form (see Fig. 1, A):

$$T = T_1 \text{ when } x < x_*(t), \ T = T_0 \text{ when } x > x_*(t). \tag{6}$$

The law of motion of the temperature front is found from the heat balance condition. We will observe the motion of this front on the infinitesimal time interval dt, having isolated the volume element $S dx_*$ traversed by the front in this time, while calculating the temperature scale from the value T_0. Equating the supply of heat from the left together with the fluid to the amount of heat required to raise a volume element of the formation from T_0 to T_1, we obtain the expression for the velocity of the thermal front

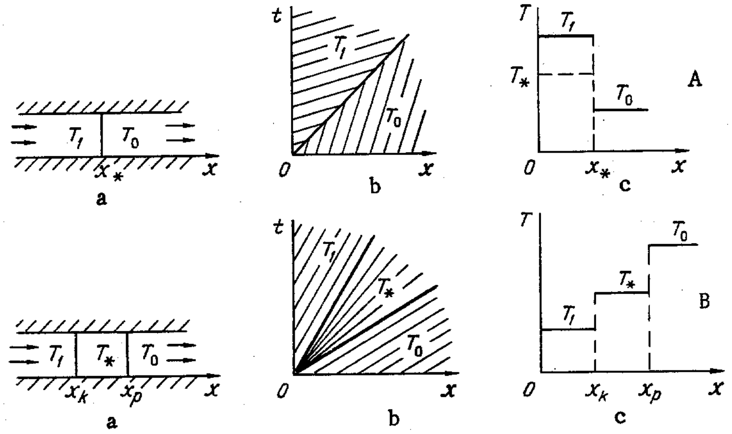

Fig. 1. Schematic representation of the solution for the injection of a hot fluid with molten impurity into a cold reservoir (A) and of a cold fluid with solid impurity into a hot reservoir (B): a) physical picture; b) family of characteristics; c) temperature profile.

$$\frac{dx_*}{dt}=\frac{c_{f1}^{p}(T_1-T_*)+c_{f1}^{k}(T_*-T_0)+\alpha_0\rho_2\vartheta}{c_*^{p}(T_1-T_*)+c_*^{k}(T_*-T_0)+m\alpha_0\rho_2\vartheta}v. \tag{7}$$

For an oil-saturated paraffin-bearing formation when $T_0 = 50$°C, $T_* = 60$°C; $T_1 = 70$°C; $m = 0.2$; $\alpha = 0.25$; $\rho_2 = 880$ kg/m³; $\vartheta = 150$ kJ/kg we obtain $\dot{x}_* = 1.54$ v. In the absence of phase transitions the velocity of the thermal front would be $v_T^k = 0.96$ v. The increase in the velocity of the thermal front is attributable to the heat released during crystallization of the paraffin. For a small temperature difference $(T_1 - T_0)$ the increase is substantial, but the velocity of the thermal front x_* always remains less than the physical velocity of the fluid v/m.

2. Let us now turn to the case of the injection of a cold fluid with a crystallized impurity into a hot porous medium occupied by the same fluid. The solution is represented schematically in Fig. 1, B.

At a distance from the inlet in the hot zone of the formation the temperature disturbances are propagated through the flow at a greater velocity than in the cold zone. Therefore between these two zones a third zone is formed, in which melting of the impurity in the injected fluid takes place. The motion of each of the fronts is determined from the characteristics of the cold and hot percolation zones, and the solution of the problem is written in the form:

$$\begin{aligned} &T = T_1 \text{ when } x < x_k = v_T^{k}t,\\ &T = T_* \text{ when } x_k < x < x_p,\\ &T = T_0 \text{ when } x > x_p = v_T^{p}t. \end{aligned} \tag{8}$$

In the impurity fusion zone where $x_k < x < x_p$ the characteristics of Eq. (3) depart from the origin to form a fan, like an expansion wave in gas dynamics [7]. To each characteristic there corresponds a certain amount of fused impurity α. The slope of the characteristic makes it possible uniquely to determine α from the equation

$$\frac{c_{f1}(\alpha)}{c_*(\alpha)}=\frac{x}{vt}, \tag{9}$$

where at the temperature T_* c_{f1} and c_* are linear functions of α and are determined from Eqs. (5) and (3).

Solving Eq. (9) for α, we obtain the explicit expression

$$\alpha = \frac{c_0(1-m)x - c_{fl}^{k}(vt - mx)}{(c_2^p - c_2^k)(vt - mx)}. \qquad (10)$$

When $x = x_p$ this expression gives $\alpha = 0$ and when $x = x_k$ it gives α_0. In the fusion zone α varies continuously from 0 to α_0.

The dimensions of the fusion zone with temerature T_* are not very great. For an oil formation with a 25% paraffin content and the above-mentioned parameters $x_k = 0.9614$ vt and $x_p = 0.9931$ vt. The impurity fusion zone represents about 3% of the percolation path $x_f =$ vt. Under actual conditions the impurity is usually inhomogeneous; the oil contains a series of paraffin fractions with different melting points. There may be one crystallization front for all the fractions but many fusion zones. In carrying out the calculations the specific heat of fusion is "smeared" over a certain temperature interval and the value of the specific heat correspondingly increased.

NOTATION

v, percolation velocity; v_f, physical velocity of the fluid in the pores; v_T, propagation velocity of the temperature disturbances; u_0, impurity particle sedimentation rate; μ, coefficient of dynamic viscosity; $\Delta\rho$, difference of the densities of the suspended particles and the fluid; R, radius of the suspended particles; g, acceleration of free fall; G, dynamic pressure gradient; m, porosity of the reservoir formation; k, its permeability; T_1, temperature of the injected fluid; T_0, temperature of the formation fluid; T_*, impurity phase transition temperature; c_*, volumetric specific heat of the formation; c_{fl}, volumetric specific heat of the fluid containing the impurity; c_1, volumetric specific heat of the fluid free of the impurity; c_2^p, volumetric specific heat of the molten impurity; c_2^k, volumetric specific heat of the solid impurity; α_0, total volumetric impurity content of the fluid; α, volumetric content of molten impurity in the fluid; ρ_1, density of the fluid; ρ_2, the density of the suspended impurity; x_*, position of the impurity crystallization front; ϑ, specific heat of fusion of the impurity; and x_k and x_p, boundaries of the impurity fusion zone.

LITERATURE CITED

1. Sh. K. Gimatudinov and A. I. Shirkovskii, Physics of Oil and Gas Reservoirs [in Russian], Moscow (1982).
2. Handbook. Oils of the USSR. Vol. 4. Oils of Central Asia, Kazakhstan, Siberia and Sakhalin Island [in Russian], Moscow (1974).
3. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics in Particulate Media, Prentice-Hall (1965).
4. V. P. Tronov, Mechanism of Formation of Resino-Paraffin Deposits and Countermeasures [in Russian], Moscow (1970).
5. M. D. Rozenberg and S. A. Kundin, Multiphase Multicomponent Percolation in Oil and Gas Production [in Russian], Moscow (1976).
6. A. B. Sheinman, G. E. Malofeev, and A. I. Sergeev, Heat Action on the Formation in the Oil Extraction Process [in Russian], Moscow (1969).
7. H. W. Liepmann and A. Roshko, Elements of Gas Dynamics, Wiley, N. Y. (1957).